Shear Strength of Soil

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Principal Stress

- \( \sigma_1 > \sigma_2 > \sigma_3 \)
- Major, Intermediate, Minor principal stress

\[
\sigma = \sigma_1 \cos^2 \alpha + \sigma_3 \sin^2 \alpha = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\alpha
\]

\[
\tau = (\sigma_1 - \sigma_3) \sin \alpha \cos \alpha = \frac{(\sigma_1 - \sigma_3)}{2} \sin 2\alpha
\]
Pole Point

- The point on the Mohr’s circle from which any extended straight line will intersect the circle at a point corresponding to stress on a plane parallel to the extended line
\[ \sigma_x, \sigma_y, \sigma_3, \sigma_1 \text{ pole} \]
Stress and Strain

\[ \sigma_1, \varepsilon_1 \]

\[ \sigma_2, \varepsilon_2 \]

\[ \sigma_3, \varepsilon_3 \]
3 Dimension

\[ \sigma_1 \neq \sigma_2 \neq \sigma_3 \neq 0 \]

\[ \varepsilon_1 \neq \varepsilon_2 \neq \varepsilon_3 \neq 0 \]
Plane Strain

\[ \sigma_1 \neq \sigma_3 \neq 0 ; \quad \sigma_2 = \sigma_x \]

\[ \varepsilon_1 \neq \varepsilon_3 \neq 0 ; \quad \varepsilon_2 = 0 \]

If the material is linearly isotropic:

\[ \sigma_2 = \nu \left( \sigma_1 + \sigma_3 \right) \]
Axisymmetric Conditions

- Axisymmetric
- Triaxial
- Uniaxial
Axisymmetric

\[ \sigma_1 \neq \sigma_3 \neq 0 ; \quad \sigma_2 = \sigma_\theta \]

\[ \varepsilon_1 \neq \varepsilon_3 \neq 0 ; \quad \varepsilon_2 = -\frac{s_r}{r} \]

$s_r =$ radial displacement
$r =$ radial coordinate of the point

For linearly isotropic materials:

\[ \sigma_2 = \nu (\sigma_1 + \sigma_3) + E \varepsilon_2 \]
Triaxial

Compression

\[ \sigma_z > \sigma_r \]
\[ \sigma_1 = \sigma_z \; ; \; \sigma_2 = \sigma_3 = \sigma_r \]
\[ \varepsilon_1 = \varepsilon_z \; ; \; \varepsilon_2 = \varepsilon_3 = \varepsilon_r \]

Extension

\[ \sigma_z < \sigma_r \]
\[ \sigma_1 = \sigma_2 = \sigma_r \; ; \; \sigma_3 = \sigma_z \]
\[ \varepsilon_1 = \varepsilon_2 = \varepsilon_r \; ; \; \varepsilon_3 = \varepsilon_z \]
Uniaxial

Same as triaxial condition except $\sigma_r = 0$

Compression

$\sigma_z > \sigma_r$ 

$\sigma_1 = \sigma_z \, ; \, \sigma_2 = \sigma_3 = \sigma_r$ 

$\varepsilon_1 = \varepsilon_z \, ; \, \varepsilon_2 = \varepsilon_3 = \varepsilon_r$
Ko Stress State

\[
\sigma_z > \sigma_r \\
\sigma_1 = \sigma_z ; \sigma_2 = \sigma_3 = \sigma_r \\
\varepsilon_1 = \varepsilon_z ; \varepsilon_2 = \varepsilon_3 = 0
\]

\[
\sigma_z < \sigma_r \\
\sigma_1 = \sigma_2 = \sigma_r ; \sigma_3 = \sigma_z \\
\varepsilon_1 = \varepsilon_2 = 0 ; \varepsilon_3 = \varepsilon_z
\]

\[
\sigma_r = \frac{\nu}{1 - \nu} \sigma_z \quad \text{If the material is linearly isotropic}
\]
<table>
<thead>
<tr>
<th>State of stress</th>
<th>Stress condition</th>
<th>Strain condition</th>
<th>Application</th>
<th>Note</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triaxial</td>
<td>None</td>
<td>None</td>
<td></td>
<td>No symmetry</td>
</tr>
<tr>
<td>Plane strain</td>
<td>$\sigma_2$ horiz.</td>
<td>$\varepsilon_2 = 0$</td>
<td>Strip foundation, retaining wall, slope</td>
<td>Same stress condition along long axis, no lateral strain</td>
</tr>
<tr>
<td>Axisymmetric</td>
<td>$\sigma_2$ horiz.</td>
<td>$\varepsilon_2 = -\frac{u}{r} \neq 0$</td>
<td>Circular foundation, pile</td>
<td>Symmetric to axis</td>
</tr>
<tr>
<td>Triaxial (Comp.)</td>
<td>$\sigma_1 = \sigma_z$</td>
<td>$\sigma_2 = \sigma_3 = \sigma_r$ $\varepsilon_2 = \varepsilon_3 = \varepsilon_r$</td>
<td>Below the center of circular tank</td>
<td>Symmetric to axis</td>
</tr>
<tr>
<td>Uniaxial (Unconfined)</td>
<td>$\sigma_1 = \sigma_z$ $\sigma_2 = \sigma_3 = 0$ $\varepsilon_2 = \varepsilon_3 = \varepsilon_r$</td>
<td>UC test</td>
<td>Symmetric to axis</td>
<td></td>
</tr>
<tr>
<td>$K_0$ state</td>
<td>$\sigma_1 = \sigma_z$ $\sigma_2 = \sigma_3 = \sigma_r$ $\varepsilon_2 = \varepsilon_3 = 0$</td>
<td>Areal fill, large tank</td>
<td>No horizontal strain, ratio of horizontal stress to vertical stress is $K_0$</td>
<td></td>
</tr>
</tbody>
</table>
Rendulic Plot

\[ \sigma_a \]

\[ \sqrt{2} \sigma_r \]
Rendulic Plot

Axial Stress $\sigma_{a'}$

Radial Stress $\sqrt{2} \sigma'_r$

$\sigma_{a'}$ $\sqrt{2} \sigma'_r$
q-p Plot

Roscoe, Schofield, and Wroth (1958)

\[ q = \sigma_1 - \sigma_3 = \bar{\sigma}_1 - \bar{\sigma}_3 \]

\[ p' = \frac{1}{3}(\bar{\sigma}_1 + \bar{\sigma}_2 + \bar{\sigma}_3) \]

Lambe and Whiman (1969)

\[ q = \frac{1}{2}(\sigma_1 - \sigma_3) = \frac{1}{2}(\bar{\sigma}_1 - \bar{\sigma}_3) \]

\[ p' = \frac{1}{2}(\bar{\sigma}_1 + \bar{\sigma}_3) \]
$\tau - \sigma$ Plot

$\tau$

$\sigma$ or $\sigma'$
Normal Stress and Shear Stress

Normal stress, $\sigma = N/A$
Shear stress, $\tau = T/A$
Stress resultant, $p = R/A$
Obliquity = $\tau/\sigma$
Angle of obliquity, $i = \tan^{-1}(\tau/\sigma)$
Coefficient of friction, $\mu = \frac{T_f}{N} = \frac{\tau_f}{\sigma}$

Friction angle, $\phi = \tan^{-1}\left(\frac{\tau_f}{\sigma}\right)$

If there is cohesion, when $\sigma = 0$, $c = \tau_f$

$\mu = \frac{\Delta T_f}{\Delta N} = \frac{\Delta \tau_f}{\Delta \sigma}$

$\phi = \tan^{-1}\left(\frac{\Delta \tau_f}{\Delta s}\right)$
Enlarged contact surfaces

$A_{\text{contact}} \ll A$

$\sigma_{\text{contact}} \gg \sigma$

Yield at contact area?
Crushing?
Jumping?
Moving over?
Ideal Shear Strength Test

- Independent control of stress and strain in each direction
- Direction of principal stress
- Rate of stress or strain application
- Uniform distribution of applied stress
- Control over pore (air or water) pressure
Parameters Obtained

- Shear strength parameters
- Pore pressure parameters
- Deformation parameters
Laboratory Shear Strength Tests

- Direct shear test
  - Ring shear
  - Simple shear, direct simple shear
- Unconfined compression test
- Triaxial test
  - Compression
  - Extension
- Plane strain test → seldom
- Dynamic strength tests
Direct Shear ASTM D3080

Normal Force, $N$

Area=$A$

Soil Specimen

Shear Force, $T$

Normal stress, $\sigma = \frac{N}{A}$

Shear stress, $\tau = \frac{T}{A}$
Stress Path

Failure envelope

Failure, continuous deformation without increasing shear stress

Shear

Consolidation
Problems with Direct Shear Test

- Nonuniform stress
  - Normal stress
  - Shear stress
- Changing area of shear plane
- Contact between soil and shear box

![Diagram showing contact between soil and shear box, squeezing of soil, and changing area of shear plane.]
Direct Shear

- **Advantages**
  - Easy to perform
  - Low cost
  - A lot of experience

- **Drawback**
  - Small-size specimens
  - Limited shear displacement
  - Forced failure plane
Ring Shear

- **Advantages**
  - Cost is not very high
  - Unlimited shear displacement

- **Drawbacks**
  - Anisotropy
  - Small specimen size
  - Forced failure plane
  - Limited experience
Unconfined Compression Test
Triaxial Test
No drainage

\[ \Delta u \]

\[ \Delta \sigma_1 \]

\[ \Delta \sigma_3 \]
\[ \Delta \sigma_3 \]

\[ \Delta u_a \]

\[ \Delta \sigma_3 \]

\[ \Delta \sigma_1 - \Delta \sigma_3 \]

\[ \Delta u_d \]

\[ \Delta u \]

\[ \Delta \sigma_1 \]

\[ \Delta \sigma_3 \]
Triaxial Compression
Triaxial Extention
Determination of Shear Strength from Test Results

- Define failure criteria
  - Peak stress difference \((\sigma_1 - \sigma_3)_{\text{max}}\)
  - Maximum stress ratio \(\left(\frac{\sigma_1 - \sigma_3}{\sigma_3}\right)_{\text{max}}\)
  - Maximum shear stress \(\tau_{\text{max}}\)
  - Specific strain level
- Determine principal stresses and shear stress
- Drained tests: 1. and 2. give same \(c, \phi\)
- Undrained tests: 1. and 2. give different sets of \(c, \phi\)
Selection of failure criteria depends on:

- Testing condition
- Field condition (Drained vs. Undrained)
- Method of analysis (Total stress vs. Effective stress)
Determining $c$ and $\phi$

- Mohr-Coulomb diagram, $\tau$ and $\sigma$
  - Tangent line of Mohr circles
- q-p’ plot
  - Linear regression of (q’, p) sets

\[
q = \frac{1}{2}(\sigma_1 - \sigma_3) = \frac{1}{2} (\bar{\sigma}_1 - \bar{\sigma}_3)
\]

\[
p' = \frac{1}{2}(\bar{\sigma}_1 + \bar{\sigma}_3)
\]
\[ q = c \cos \phi + p' \sin \phi \]

\[ \tan \psi = \sin \phi \]

\[ d = c \cos \phi \]

\[ p' \frac{1}{2} (\bar{\sigma}_1 + \bar{\sigma}_3) \]
Unconsolidated Undrained Test

- UU test or Q (quick) test $\sigma_1 = 0$

\[ \begin{align*}
\sigma_3 &= 0 \\
\Delta u_1 &= \frac{\Delta \sigma_3}{\sigma_3} \\
\Delta \sigma_3 &= \Delta u_2
\end{align*} \]

\[ \sigma_{31} = \sigma_{30} + (\Delta \sigma_3 - \Delta u_1) = \sigma_{30} = -u_0 \]
Reason for the Confining Stress

- More similar to actual field condition where all around pressure exists
- Increase the degree of saturation
- Can make the fissures (formed during the extrusion of tube samples) in the specimens close up
$\overline{\sigma}_{3f} = \sigma_3 - \Delta u_1 - \Delta u_2 - u_0$
\[ \sigma_3 = \sigma_{3c} - \mu_c \]
Consolidated Drained Test

- CD test or S (slow) test

\[ \sigma_3 = 0 \]

\[ \Delta u = 0 \]

\[ u_c = \text{back pressure} \]

\[ \sigma_{3c} = \sigma_1 - \sigma_3 \]
\[
\sigma_3 = \sigma_{3c} - u_c \\
\bar{\sigma}_3 = \sigma_3 - \Delta u
\]
CD, S Tests Stress Path

\[(\sigma_1 - \sigma_3)\]

Effective stress failure envelope

failure

\(\sigma_3\) or \(\bar{\sigma}_3\)
\[
\sigma_{3f} = \sigma_3 - u_c
\]

\[u_c = \text{backpressure}\]
Consolidated Undrained Test

- Without pore pressure measurement → CU test or R (rapid) test
- With pore pressure measurement → CU or R test
- Isotropic consolidation → CIU
- Anisotropic consolidation → CAU
\[ \sigma_3 = 0 \]

\[ u_0 \xrightarrow{\sigma_3 = 0} u_c \]

\[ u_c = \text{back pressure} \]

\[ \Delta u_2 \]

\[ \Delta u_3 \]

\[ \sigma_3 = 0 \]

\[ \Delta \sigma_3 \]

\[ \sigma_1 - \sigma_3 \]
Pore Pressure Coefficients

- In an undrained test, the change of pore pressure can be related to the changes of stresses as:

\[
\Delta u = B[\Delta \sigma_3 + A(\Delta \sigma_1 - \Delta \sigma_3)]
\]

\[A \text{ and } B \text{ are the pore pressure coefficients}\]

\[
\Delta u_a = B \Delta \sigma_3
\]
Change of total volume:

$$\Delta V_c = -c_c \cdot V \cdot (\Delta \sigma_3 - \Delta u_a) = -c_c \cdot V \cdot \Delta \sigma_3$$

Change of volume of voids:

$$\Delta V_v = -c_v \cdot nV \cdot \Delta u_a$$

$c_c =$ compressibility of the soil skeleton

$c_v =$ compressibility of the fluid in voids

$$\Delta V_c = \Delta V_v$$
\[ B = \frac{1}{1 + \frac{n \cdot c_v}{c_c}} \]

For saturated soils, if \( c_c \gg c_v \), \( B \to 1 \) (soil skeleton is much more compressible than the fluid)

For very stiff clays whose \( c_c \) is also very small, \( B \) may not be close to 1 even if the soil is almost saturated.
● $A$ and $B$ can only be determined from test results

● When $(\sigma_1 - \sigma_3)$ remains 0, $B$ can be determined from the relationship between the increasing $\sigma_3$ and $u$

   ● For saturated specimens, $B$ is close to 1

● Get $A$ by from the changes of $u$ and $(\sigma_1 - \sigma_3)$

● $A$ is not a constant, it changes through out the test

\[
A = \frac{\Delta u_d}{\Delta \sigma_1 - \Delta \sigma_3} \cdot \frac{1}{B}
\]
Elastic Material

\[\Delta \varepsilon_1 = \frac{1}{E} \left[ \Delta \sigma_1 - \nu (\Delta \sigma_2 + \Delta \sigma_3) \right]\]

\[\Delta \varepsilon_2 = \frac{1}{E} \left[ \Delta \sigma_2 - \nu (\Delta \sigma_1 + \Delta \sigma_3) \right]\]

\[\Delta \varepsilon_3 = \frac{1}{E} \left[ \Delta \sigma_3 - \nu (\Delta \sigma_1 + \Delta \sigma_2) \right]\]
If saturated $\rightarrow$ no volume change:

$$\Delta \varepsilon_v = \Delta \varepsilon_1 + \Delta \varepsilon_2 + \Delta \varepsilon_3 = 0$$

Rewrite as effective stresses:

$$\frac{1}{E} [\Delta \bar{\sigma}_1 + \Delta \bar{\sigma}_2 + \Delta \bar{\sigma}_3 - 2\nu(\Delta \bar{\sigma}_1 + \Delta \bar{\sigma}_2 + \Delta \bar{\sigma}_3)] = 0$$

$$\frac{1}{E} [(1 - 2\nu)(\Delta \bar{\sigma}_1 + \Delta \bar{\sigma}_2 + \Delta \bar{\sigma}_3)] = 0$$
\[ \nu \neq 0.5 \]

\[ \Delta \sigma_2 = \Delta \sigma_3 \]

\[ \Delta \sigma_1 + 2\Delta \sigma_3 = 0 \]

\[ \Delta \sigma_1 = \Delta \sigma_1 - \Delta u \]

\[ \Delta \sigma_3 = \Delta \sigma_3 - \Delta u \]

\[ \Delta \sigma_1 + 2\Delta \sigma_3 - 3\Delta u = 0 \]
\[ 3\Delta \sigma_3 + (\Delta \sigma_1 - \Delta \sigma_3) = 3\Delta u \]

\[ \Delta u = \Delta \sigma_3 + \frac{1}{3} (\Delta \sigma_1 - \Delta \sigma_3) \]

\[ A = \frac{1}{3} \]
Use $B$ to check the saturation of soil specimens.

- Incremental increase of $\Delta \sigma_3 \rightarrow$ check the increase of $\Delta u \rightarrow$ compute $B$
- If $B$ is close to 1, the specimen is near saturation
Although $B$ is an indication of the degree of saturation, there is no way of knowing the degree of saturation from values of $B$.
\[ \Delta u = B[\Delta \sigma_3 + A(\Delta \sigma_1 - \Delta \sigma_3)] \]

\[ \Delta u = B\Delta \sigma_3 + A \cdot B(\Delta \sigma_1 - \Delta \sigma_3) \]

\[ \bar{A} = A \cdot B \]

For \( B = 1 \)

\[ \bar{A} = \frac{\Delta u}{\Delta \sigma_1 - \Delta \sigma_3} \]

During shearing \( \Delta \sigma_3 = 0 \)

\[ \bar{A} = \frac{\Delta u}{\Delta \sigma_1} \]
- $A$ depends on:
  - the state of the specimen (soil type),
  - the stress history (overconsolidation),
  - the stress state (anisotropy),
  - the stress level,
  - loading conditions (loading or unloading), and
  - other conditions
- $A$ can vary from negative values to greater than 1
- $A$ at failure is defined as $A_f \rightarrow \overline{A}_f$
Back Pressure

- Increase the degree of saturation
- Increase pore water pressure, $u$, in the specimen while maintaining the effective confining (consolidation) stress $\sigma_3$
- For example:

<table>
<thead>
<tr>
<th>$\sigma_3$</th>
<th>100 kPa</th>
<th>150 kPa</th>
<th>200 kPa</th>
<th>250 kPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>50 kPa</td>
<td>100 kPa</td>
<td>150 kPa</td>
<td>200 kPa</td>
</tr>
<tr>
<td>$\sigma_3$</td>
<td>50 kPa</td>
<td>50 kPa</td>
<td>50 kPa</td>
<td>50 kPa</td>
</tr>
</tbody>
</table>

Check $B$ at every stage
How Does Backpressure Works?

- Elevated pore water pressure can squeeze on the trapped air bubbles and make them decrease in size.
- Elevated pore water pressure would make the trapped air more likely to dissolve in water.
- Some would apply a small hydraulic gradient to help flush out the air during the saturation process.
\[ \overline{\sigma}_{3f} = \sigma_3 - \Delta u_1 - \Delta u_2 - u_0 \]
Effective stress path for undrained test, \( u \) decreases

Total stress failure envelope

Effective stress failure envelope

Eff. stress path for undrained test, \( u \) decreases

\( (\sigma_1 - \sigma_3) \)

\( \sigma_3 \) or \( \bar{\sigma}_3 \)
Increased $u$

$\left( \sigma_1 - \sigma_3 \right)$

Effective stress path for undrained test
$u$ increases

Effective stress failure envelope

Total stress failure envelope

$\sigma_3$ or $\bar{\sigma}_3$
Decreased $u$

$(\sigma_1 - \sigma_3)$

Total stress failure envelope

Effective stress failure envelope

Eff. stress path for undrained test, $u$ decreases

$\sigma_3$ or $\overline{\sigma}_3$
CU, R Tests

$$\sigma_1 - \sigma_3$$

High sensitivity

Low sensitivity

Effective stress path for undrained test $u$ increases

$$\sigma_3 \text{ or } \bar{\sigma}_3$$
UU, Q Tests (CU, R Tests)

\[(\sigma_1 - \sigma_3)\] 

\[(\sigma_1 - \sigma_3)_{\text{max}} \rightarrow \text{Failure}\] 

R envelope 

Q envelope
Energy Corrections

- Direct shear tests
- Triaxial tests
Direct Shear Tests

\[ \Delta v > 0 \text{ for expansion} \]

Total work done by applied shear stress

\[
\begin{align*}
&= \tau \cdot A \cdot d\Delta h \\
&= \tau^* \cdot A \cdot d\Delta h + \sigma \cdot A \cdot d\Delta v
\end{align*}
\]

Energy used to expand (+) or contract (-) against normal stress \( \sigma \)

Shear stress used to shear the soil to failure, \( \tau^* \):

\[
\tau^* = \tau - \sigma \frac{d\Delta v}{d\Delta h}
\]
Triaxial Tests

- Work done by axial stress: \( \sigma_1 \cdot d\varepsilon_1 \)

- Work done to expand the specimen laterally against the confining stress:
  \[-\sigma_3 \cdot d\varepsilon_3 - \sigma_3 \cdot d\varepsilon_3\]

- \( dW = \) work required to shear the specimen (subject to \( \sigma_3 d\varepsilon_3 \))
\[ \sigma_1 \cdot d \varepsilon_1 = dW - \sigma_3 \cdot d \varepsilon_3 - \sigma_3 \cdot d \varepsilon_3 \quad \rightarrow \text{Dilate if } d \varepsilon_3 > 0 \]

\[ (\sigma_1 - \sigma_3) d \varepsilon_1 = dW - \sigma_3 (d \varepsilon_3 + d \varepsilon_3 + d \varepsilon_1) \]

Volumetric strain \( = d \varepsilon_v = d \varepsilon_1 + d \varepsilon_2 + d \varepsilon_3 \)

Bishop assumed:

\[ dW = (\sigma_1 - \sigma_3)^* \cdot d \varepsilon_1 \]

\[ (\sigma_1 - \sigma_3) d \varepsilon_1 = (\sigma_1 - \sigma_3)^* d \varepsilon_1 - \sigma_3 d \varepsilon_v \]
\[(\sigma_1 - \sigma_3)^* = (\sigma_1 - \sigma_3) + \sigma_3 \frac{d\varepsilon_v}{d\varepsilon_1}\]

Say \(c = 0\)

\[
\sin \phi = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3} \quad \quad \sin \phi = \frac{(\sigma_1 - \sigma_3)}{(\sigma_1 - \sigma_3) + 2\sigma_3}
\]

\[
\sin \phi^* = \frac{(\sigma_1 - \sigma_3)^*}{(\sigma_1 - \sigma_3)^* + 2\sigma_3}
\]
Measured $\phi$

Dilatancy effect

Particle orientation effect

$\phi_{cv}$: constant volume

$\phi_u$ = mineral-to-mineral friction angle

Initial porosity, $n_i$

46% Loose

34% Dense

$\phi_u = \text{mineral-to-mineral friction angle}$
Bishop

\[
\sin \phi^* = \frac{\sigma_1}{\sigma_3} + 1 - \left(1 - \frac{d\varepsilon_v}{d\varepsilon_a}\right)
\]

Rowe

\[
\sin \phi^* = \frac{\sigma_1}{\sigma_3} + \left(1 - \frac{d\varepsilon_v}{d\varepsilon_a}\right) - \left(1 - \frac{d\varepsilon_v}{d\varepsilon_a}\right)
\]

If soil tends to dilate, \( \phi^*_{\text{Rowe}} < \phi^*_{\text{Bishop}} \)
(Rowe considers particle rearrangement effect)
Effect of Energy Corrections

- S test
- Uncorrected $\phi = 42^\circ$
- Bishop’s correction $\phi = 38^\circ \rightarrow$ more appropriate
- Rowe’s correction $\phi = 24^\circ$
Effect of Confining Pressure

\[ \bar{c} = 0 \quad \bar{\phi} = f \left( \frac{\bar{\sigma}_1}{\bar{\sigma}_3} \right) \]

Pourse and Bell (1971) – sand, D_r=94%

<table>
<thead>
<tr>
<th>( \bar{\sigma}_3 )</th>
<th>( \bar{\phi} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2 psi</td>
<td>52°</td>
</tr>
<tr>
<td>10 psi</td>
<td>28.5°</td>
</tr>
</tbody>
</table>
Lee and Seed (1967) – Sacramento River sand

\[ \sigma_3 \quad \phi \]

\[ \text{Dr} = 100\% \quad 40 \text{ ksc} \quad 42^\circ \]

\[ 140 \text{ ksc} \quad 33^\circ \quad \text{Some particle breakage} \]

\[ \sigma_3 \quad \phi \]

\[ \text{Dr} = 38\% \quad \text{Low stress level} \quad 34^\circ \]

\[ \text{High stress level} \quad 32^\circ \]

Before shear 5% passed through #200 sieve
After shearing at 140 ksc, 50% passed through #200 sieve
Dense Loose

\[ \tau \approx 40 \text{ ksc} \]
Measured $\phi$

Dilatancy effect

Particle orientation effect

$\phi_{cv}$: constant volume

$\phi_u$

$\bar{\sigma}_3$
Some Other “Corrections” or “Considerations”

- Measured stress
  - Membrane correction
  - Effect of friction between soil specimen and filter paper/porous disk/cap and pedestal

- B value
  - Flexibility of drainage tubing
  - Use metal tubing
Stress-Strain Properties

- At low to medium confining pressure

\[ \frac{\sigma_1}{\sigma_3} \]

- Dense sand

- Loose sand

\( \varepsilon_v \) expansion

\( \varepsilon_a \) contraction

\( \varepsilon_v \approx 20\% \)

\( \varepsilon_a \approx 4\% \)
At high confining stress

- \( \frac{\sigma_1}{\sigma_3} \)
  - \( \varepsilon_v \) expansion
  - \( \varepsilon_a \) contraction

Dense sand: ~20%

Loose sand: ~4%
- At larger strains dense sand and loose sand tend to have the same void ratio →
- They tend to have the same strength
Critical Void Ratio

- A. Casagrande

Drained shear

Void ratio, $e$

Loose sand
Contraction

Where shearing starts

Critical void ratio line

Dense sand
Dilate

Where shearing starts

Confining stress
Undrained shear

Void ratio, $e$

Change of pore water pressure

Where shearing starts

Higer shear strength

Dense sand

Critical void ratio line

Confining stress

Loose sand

Where shearing starts
**Dense Sand**

\( \bar{R} \) test, \( \Delta V = 0 \)

- Increase: \( \frac{\sigma_1}{\sigma_3} \)
- Decrease: \( \sigma_1 - \sigma_3 \)

\( \sim 4\% \) to \( \sim 20\% \)
Loose Sand

$\bar{R}$ test, $\Delta V = 0$

$\sigma_1 - \sigma_3$

$\frac{\sigma_1}{\sigma_3}$

$\varepsilon_a$

increase

u

decrease

$\sim 4\%$

$\sim 20\%$
For normally consolidated clay, when \((\sigma_1 - \sigma_3)\) reaches maximum, its shear strength has not been fully mobilized yet → the shear strength is still increasing.

For overconsolidated clay, although the shear strength has been fully mobilized, \((\sigma_1 - \sigma_3)\) keeps increasing due to the increase of effective confining stress → the difference is not as significant as for N.C. clay.
\((\sigma_1 - \sigma_3)\)

Stress path tangency

\[\left(\frac{\sigma_1}{\sigma_3}\right)_{\text{max}}\]

Dense sand, OC clay

Loose sand, NC clay
Effive and total stress path of drained test

Total stress path for undrained test

Eff. stress path for undrained test

$u$ decreases

Effective stress path for undrained test

$u$ increases

Stress path tangency

Effive and total stress path of drained test

Total stress path for undrained test
Normally Consolidated and Lightly Overconsolidated Clay
Loose Sand

\[(\sigma_1 - \sigma_3)_{\text{max}}\]

\[\left(\frac{\sigma_1 - \sigma_3}{\sigma_3}\right)_{\text{max}}\]
Overconsolidated Clay

Dense Sand

\[ \left( \sigma_1 - \sigma_3 \right)_{\text{max}} \]

\[ (\sigma_1 - \sigma_3) \]

\[ \frac{\left( \sigma_1 - \sigma_3 \right)}{\sigma_3} \]_{\text{max}}

\[ \varepsilon_a \]
Steady-State Strength

- The steady-state deformation for any mass of particles is that the state in which the mass is continuously deforming at constant volume, constant normal effective stress, constant shear stress, and constant velocity.
Grain Size

- Diameter of specimens should be at least $6 \times$ (largest particle size)
- Can use specimens with the same shape of particle distribution curve for testing
- Also they have to have the same relative density
- $\phi$ increases as average effective stress increases and as max. particle size decreases
Effect of State of Stress

\[ \phi \sim 4^\circ \]

Plane strain

Triaxial

Initial porosity, \(n_i\)

Dense

Loose
Direct Shear Test

- Direct shear is a “plane strain” test → higher friction angle
- The stress on the failure plane is not uniform, not all parts reached peak stress at “failure”
• $\varepsilon_{f, tx}$ is about $3 \times \varepsilon_{f, ps}$
• $\phi_{f, TC}$ is about the same as $\phi_{f, TE}$

\begin{align*}
(\sigma_1 - \sigma_3) & \quad \text{Dense sand} \\
\sim 1.5\% & \quad \sim 4\% \\
\text{Plane strain} & \quad \text{Triaxial} \\
\end{align*}

\begin{align*}
(\sigma_1 - \sigma_3) & \quad \text{Loose sand} \\
\text{Plane strain} & \quad \text{Triaxial} \\
\varepsilon_a & \\
\varepsilon_a
\end{align*}