Well Hydraulics
Basic Assumptions

- The aquifer is bounded on the bottom by a confining layer.
- All geologic formations are horizontal and of infinite horizontal extent.
- The potentiometric surface of the aquifer is horizontal prior to the start of the pumping.
- The potentiometric surface of the aquifer is not changing with time prior to the start of the pumping.
All changes in the position of the potentiometric surface are due to the effect of the pumping well alone.

The aquifer is homogeneous and isotropic.

All flow is radial toward the well.

Ground-water flow is horizontal.
Darcy’s law is valid

Ground water has a constant density and viscosity

The pumping well and the observation wells are fully penetrating, i.e., they are screened over the entire thickness of the aquifer

The pumping well has an infinitesimal diameter and is 100% efficient
Groundwater Wells

- Production Wells
- Injection Wells
- Remediation Wells
  - Pumping wells
  - Injection wells
Wells in Confined and Unconfined Aquifers

- In unconfined aquifers, pumping will result in drawdown of the water table.
- In confined aquifers, pumping will cause drawdown of the potentiometric surface.
  - All pores in the confined aquifer will still be saturated.
Pumping Rate

- The rate of which water is extracted from the pumping well
- Usually written as $Q$
Drawdown

- Lowering of water table caused by pumping of wells
- Defined as the difference between elevations of the current water table and water table before pumping began
Cone of Depression

- Cone of depression will form in the aquifer around a pumping well as the water level declines
Computing Drawdown Caused by a Pumping Well

- Unsteady radial flow
- Radial symmetry

Point in a plane

\( r \)

\( \theta \)

Origin

Polar axis
General Procedure

\[
\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = \frac{S}{T} \frac{\partial h}{\partial t} \quad \text{Conditions Assumptions Confined?}
\]

\[
\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} = \frac{S}{T} \frac{\partial h}{\partial t}
\]

Drawdown, \( s \):

\[
s = \frac{Q}{4\pi T} \int_u^\infty \frac{e^{-u}}{u} \, du = \frac{Q}{4\pi T} W(u)
\]
Steady Flow to a Well in a Confined Aquifer

- Homogeneous
- Fully penetrating well
- Thickness $B$
- Transmissivity $T$
Fig. 7.2  Fully penetrating well pumping from a confined aquifer
At face 1

\[ Q = -KA \frac{\partial h}{\partial r} \]

\[ A_1 = z\pi \ r_1 \ b \]

\[ Q = -K \ z\pi \ r_1 b \left( \frac{\partial h}{\partial r} \right)_1 \]

Gradient Only Toward Well

Horizontal Flow Fully Penetrating Homog.
Isotropic

Same K all Around Well

Steady Flow
\[ Q_1 - Q_2 = 2\pi \left[ \left( r \frac{\partial h}{\partial r} \right)_2 - \left( r \frac{\partial h}{\partial r} \right)_1 \right] \]

\[ = 2\pi \ T \left[ r \frac{\partial^2 h}{\partial r^2} + \frac{\partial h}{\partial r} \right] \Delta r \]

Rate of Storage Accumulation in an Element of width \( \Delta r \)

\[ Q \rightarrow \frac{dV}{dt} = S \ A_{surf} \ \frac{\partial h}{\partial t} = S \ (2\pi \ r\Delta r) \frac{\partial h}{\partial t} \]
\[
\left( r \frac{\partial h}{\partial r} \right)_2 - \left( r \frac{\partial h}{\partial r} \right)_1 = \frac{\partial}{\partial r} \left( r \frac{\partial h}{\partial r} \right) \Delta r
\]

\[
\frac{\partial r}{\partial r} \frac{\partial h}{\partial r} + r \frac{\partial^2 h}{\partial r^2}
\]
Differences in Flow Between Faces $r_1$ and $r_2$

\[
Q_1 = 2\pi (Kb) r_1 \left( \frac{\partial h}{\partial r} \right)_1
\]

\[
Q_2 = 2\pi (Kb) r_2 \left( \frac{\partial h}{\partial r} \right)_2
\]

\[
Q_1 - Q_2 = 2\pi T \left[ \left( r \frac{\partial h}{\partial r} \right)_2 - \left( r \frac{\partial h}{\partial r} \right)_1 \right]
\]
Applying Continuity Equation:

\[ Q_1 - Q_2 = \text{Rate of Storage Accum.} \]

\[
= 2\pi \ T \left[ r \left( \frac{\partial^2 h}{\partial r^2} + \frac{\partial h}{\partial r} \right) \right] \Delta r = S(2\pi \ r \Delta r) \frac{\partial h}{\partial t} \\
= \frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} = \frac{S}{T} \frac{\partial h}{\partial t}
\]

Assumptions
- Instant. Release. from S
- No Background Gradient

Horizontal Flow
Magic ground Water Function

\[ h = \frac{v}{4\pi Tt} e^{-\left(\frac{Sr^2}{4Tt}\right)} = \frac{Q}{4\pi T} e^{-u} \]

Familiar The r’s Equation

\[ s = \frac{Q}{4\pi T} \int_{u}^{\infty} e^{-x} \frac{dx}{x} \]

Does it satisfy differential equation?

\[ \frac{\partial h}{\partial r} = \frac{v}{4\pi Tt} e^{-\left(\frac{sr^2}{4Tt}\right)} \left( -\frac{2Sr}{4Tt} \right) \]
Proved that if

\[ h = \frac{\nu}{4\pi Tt} e^{-u} \]

then

\[ \frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} = \frac{S}{T} \frac{\partial h}{\partial t} \]

Will be true

One soln to partial differential equation
\[ T \left( \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} \right) = S \frac{\partial h}{\partial t} \]

Steady flow or negligible \( S \) \( \Rightarrow \)

\[ \nabla^2 h \equiv \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0 \]
Equation for confined flow

\[ \frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} = \frac{S}{T} \frac{\partial h}{\partial t} \]

- \( h \) is hydraulic head (L)
- \( S \) is storativity (dimensionless)
- \( T \) is transmissivity (L²/T)
- \( t \) is time (T)
- \( r \) is radial distance from the pumping well (L)
If there is leakage through a confining layer, or recharge to the aquifer, then:

$$\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} + \frac{e}{T} = \frac{S \partial h}{T \partial t}$$

- $e$ is the rate of vertical leakage (L/T)
Equating inflow, $Q$, to outflow $Q_w$:

$$Q = A \times q_r = 2\pi r B \times K \frac{\partial h}{\partial r} = Q_w = \text{constant}$$

- $q_r$ is the specific recharge in the radial direction
By integrating, between \( r = r_w \) and \( h(r_w) = h_w \), and \( r \rightarrow \infty \),

\[
h(r) - h_w = \left( \frac{Q_w}{2\pi T} \right) \ln\left( \frac{r}{r_w} \right)
\]

Valid only in the close proximity of a well where steady flow has been established.
By integrating from $r_w$ to $R$, drawdown $s_w$ at $r$ is given by:

$$s_w = H - h_w = h(R) - h(r_w) = \left(\frac{Q_w}{2\pi T}\right) \ln\left(\frac{R}{r_w}\right)$$

Between any two distances $r_1$ and $r_2$ ($> r_1$)

[Thiem equation (1906)]

$$h(r_2) - h(r_1) = s(r_1) - s(r_2) = \left(\frac{Q_w}{2\pi T}\right) \ln\left(\frac{r_2}{r_1}\right)$$
Between any two distances $r$ and $R$

$$s(r) = h(R) - h(r) = \left(\frac{Q_w}{2\pi T}\right) \ln\left(\frac{R}{r}\right)$$

The shape of the curve $h=h(r)$, given $h_w$ and $H$ at $r_w$ and $R$, is independent of $Q_w$ and $T$

$$h(r) - h_w = (H - h_w) \frac{\ln(r/r_w)}{\ln(R/r_w)}$$
Flow in a Completely Confined Aquifer

- The aquifer is confined top and bottom.
- There is no source or recharge to the aquifer.
- The aquifer is compressible and water is released instantaneously from the aquifer as the head is lowered.
- The well is pumped at a constant rate.
The Theis, or nonequilibrium, equation:

\[ H - h = \frac{Q_w}{4\pi T} \int_{u}^{\infty} \frac{e^{-u}}{u} \, du \]

\[ H - h = \frac{Q_w}{4\pi T} \left[ -0.5772 - \ln u + u - \frac{u^2}{2 \cdot 2!} + \frac{u^3}{3 \cdot 3!} - \frac{u^4}{4 \cdot 4!} + \cdots \right] \]

\[ u = \frac{r^2 S}{4Tt} \]
Flow in a Leaky, Confined Aquifer

\[
\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} - \frac{(h_0 - h)K'}{Tb'} = S \frac{\partial h}{\partial t}
\]

- \(K'\) is the vertical hydraulic conductivity of the leaky layer
- \(b'\) is the thickness of the leaky layer
Fig. 7.3 Fully penetrating well in an aquifer overlain by a semipermeable confining layer
Comparison of Drawdown in Different Confined Aquifer

I  No leakage (Fig. 7.4)

II Leakage without storage in a finite semipervious layer

III Leakage with storage in an infinite semipervious layer

IV Leakage with storage in a finite semipervious layer
Fig. 7.4 Plots of dimensionless drawdown as a function of time for an aquifer with various types of overlying confining layer.
Flow in an Unconfined Aquifer

\[ K_r \frac{\partial^2 h}{\partial r^2} + K_r \frac{1}{r} \frac{\partial h}{\partial r} + K_v \frac{\partial^2 h}{\partial z^2} = S \frac{\partial h}{\partial t} \]

- \( K_r \) is radial hydraulic conductivity
- \( K_v \) is vertical hydraulic conductivity
Newman’s Solution

Assumption

- The aquifer is unconfined
- The vadose zone has no influence on the drawdown
- Water initially pumped comes from the instantaneous release of water from elastic storage
Eventually water comes from storage due to gravity drainage of interconnected pores.

The drawdown is negligible compared with the saturated aquifer thickness.

The specific yield is at least 10 times the elastic storativity.

The aquifer may be – but does not have to be – anisotropic with the radial hydraulic conductivity different than the vertical hydraulic conductivity.
Newman’s Solution

\[ H - h = \frac{Q_w}{4\pi T} W(u_A, u_B, \Gamma) \]

- Where \( u_A \) is the well function for the water-table aquifer.
\[ u_A = \frac{r^2 S}{4Tt} \] (for early drawdown data)

\[ u_B = \frac{r^2 S_y}{4Tt} \] (for later drawdown data)

\[ \Gamma = \frac{r^2 K_v}{b^2 K_h} \]

- \( b \) is the initial thickness of aquifer
- \( S \) is the storativity
- \( S_y \) is the specific yield
Steady Flow to a Well in a Unconfined Aquifer

- The flow is radially symmetric between circular equipotential boundaries at $r=R$ and $r=r_w$.
- The potential distribution $h=h(r,z)$ satisfies the continuity equation:

\[
\frac{\partial^2 h}{\partial r^2} + \frac{1}{r} \frac{\partial h}{\partial r} + \frac{\partial^2 h}{\partial z^2} = 0
\]
Determining Aquifer Parameters from Time-Drawdown Data

Assumptions:

- The pumping well is screened only in the aquifer being tested
- All observation wells are screened only in the aquifer being tested
- The pumping well and the observation wells are screened throughout the entire thickness of the aquifer
Steady-State Conditions

- **Steady-state** → No further drawdown with time
- The cone of depression stops growing because it has reached a recharge boundary
- The hydraulic gradient of the cone of depression causes water to flow at a constant rate from the recharge boundary to the well
- Radial symmetry → the recharge boundary has an unlikely circular geometry centered about the pumping well
Steady Radial Flow in a Confined Aquifer

Assumptions:

- The aquifer is confined at the top and bottom
- The well is pumped at a constant rate
- Equilibrium has been reached; i.e., there is no further change in drawdown with time
Fig. 7.5 Equilibrium drawdown: A. confined aquifer;
B. unconfined aquifer
From Darcy’s law:

\[ Q = (2\pi rb)K\left(\frac{dh}{dr}\right) \]

\[ Q = 2\pi rT\left(\frac{dh}{dr}\right) \]

\[ dh = \frac{Q}{2\pi T} \frac{dr}{r} \]
\[ \int_{h_1}^{h_2} dh = \frac{Q}{2\pi T} \int_{r_1}^{r_2} \frac{dr}{r} \]

\[ h_2 - h_1 = \frac{Q}{2\pi T} \ln \left( \frac{r_2}{r_1} \right) \]

\[ T = \frac{Q}{2\pi (h_2 - h_1)} \ln \left( \frac{r_2}{r_1} \right) \]
Steady Radial Flow in an Unconfined Aquifer

Assumptions:

- The aquifer is unconfined and underlain by a horizontal aquiclude
- The well is pumped at a constant rate
- Equilibrium has been reached; i.e., there is no further change in drawdown with time
From Darcy’s law:

\[ Q = (2\pi rh) K \left( \frac{dh}{dr} \right) \]

\[ hdh = \frac{Q}{2\pi K} \frac{dr}{r} \]
\[ \int_{h_1}^{h_2} h dh = \frac{Q}{2\pi K} \int_{r_1}^{r_2} \frac{dr}{r} \]

\[ h_2^2 - h_1^2 = \frac{Q}{\pi K} \ln \left( \frac{r_2}{r_1} \right) \]

\[ K = \frac{Q}{\pi (h_2^2 - h_1^2)} \ln \left( \frac{r_2}{r_1} \right) \]
Nonequilibrium Flow Conditions

- Many aquifer tests will never reach equilibrium
- The cone of depression will continue to grow with time
- Also known as *transient* condition
Analysis of transient time-drawdown data from an observation well can be used to determine both the **transmissivity** and the **storativity** of an aquifer.

No observation well → transient time-drawdown data from the pumping well can be used to determine the **transmissivity** but not the **storativity** of an aquifer.
Nonequilibrium Radial Flow in a Confined Aquifer – Theis Method

\[ T = \frac{Q}{4\pi(H-h)} W(u) \]

- \( W(u) \) is the well function of \( u \)
Theis Type Curve

- Theis developed a graphical means of solution to the Theis equations
- Plot of $W(u)$ as a function of $1/u$ on full logarithmic paper
- The graph has the shape of cone of depression near the pumping well
- The curve is known as Theis type curve or Reverse type curve
Fig. 7.6 The nonequilibrium reverse type curve for a fully confined aquifer
Fig. 7.7 Field-data plot on logarithmic paper for Theis curve-matching technique
Fig. 7.8  Match of field-data plot to Theis type curve
- Plot drawdown (log) vs. time (log)
- Match the plot with type curve
- Find a match point (arbitrary)
- Obtain \( W(u), 1/u, (H-h_0) \) from match point
- Substitute values of \( Q, (H-h_0), W(u) \) from the match point into equations to find \( T \) and \( S \)
After pumping for some time, $u$ becomes small

If $u$ gets less than 0.5 → ignore all higher powers of $u$

$$H - h = \frac{Q_w}{4\pi T} \left[-0.5772 - \ln u + u - \frac{u^2}{2 \cdot 2!} + \frac{u^3}{3 \cdot 3!} - \frac{u^4}{4 \cdot 4!} + \cdots \right]$$
becomes

\[ H - h = \frac{Q_w}{4\pi T} [-0.5772 - \ln u] \]

or

\[ T = \frac{Q_w}{4\pi (H - h)} [-\ln(1.78) - \ln\left(\frac{r^2 S}{4Tt}\right)] \]

\[ T = \frac{2.3Q_w}{4\pi (H - h)} \log\left(\frac{2.25Tt}{r^2 S}\right) \]
- Plot drawdown vs. time (log)
- Draw a straight line through field data points and extend backward to the zero drawdown axis.
- Get $t_0$ and the slope of the straight line $\Delta(H-h)$ per log cycle
\[ T = \frac{2.3Q_w}{4\pi\Delta(H - h)} \]

\[ S = \frac{2.25Tt_0}{r^2} \]
Fig. 7.9 Jacob method of solution of pumping-test data for a fully confined aquifer. Drawdown is plotted as a function of time on the semilogarithmic paper.
Nonequilibrium Radial Flow in an Unconfined Aquifer

- Two sets of type curves
- Type-A curves are good for early drawdown data
- Type-B curves are used for later drawdown data, when effects of gravity drainage are becoming smaller
- Type-B curves end on a Theis curve
Fig. 7.15  Type curves for drawdown data from fully penetrating wells in an unconfined aquifer
Effect of Partial Penetration of Wells

- If two observation wells equidistant from the pumping well are screened in different parts of the aquifer, the time-drawdown curves may be different.

- Depending upon the length and relative position of observation-well screens, it is possible for a more distant well to have a greater drawdown than a closer well.
Fig. 7.17  Flow lines toward a partially penetrating well in a confined aquifer
The effects of partial penetration produce a time-drawdown curve similar in shape to one produced when there is a drawdown leakage from storage through a thick, semipervious layer.

Partial-penetration effects may produce a time-drawdown curve that resembles the effect of a recharge boundary, a fully penetrating well in either a sloping water-table aquifer or an aquifer of nonuniform thickness.